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LETTER TO THE EDITOR

Length scaling in MFRG

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Abstract. The mean-field renormalisation group (MFRG) has so far given poor estimates for critical indices. It is argued that this can be traced to an unsatisfactory definition of the length scaling factor. A new definition, that improves the estimates in most cases, is proposed.

The mean-field renormalisation group (MFRG) of Indekeu *et al* (1982) has proven to be a powerful method for determination of phase diagrams and critical properties of spin systems. This includes both classical (Indekeu *et al* 1982, Slotte 1984, de Alcantara Bonfim 1984, de Alcantara Bonfim and Sá Barreto 1985, Plascak and Sá Barreto 1986) and quantum (Indekeu *et al* 1982, Plascak 1984a, b) systems. The spirit of the method is to compare the magnetisation (order parameter) of two clusters of different sizes. The clusters are subjected to a symmetry breaking field (a mean field) at the edges. Finite-size scaling (Fisher 1971, Suzuki 1977, Barber 1983) is assumed both for the magnetisation, m, and for the mean field, μ. If the expansion of the magnetisations to first order in the mean field is

$$\begin{aligned} m_N &= A_N(K)\mu_N \\ m_{N'} &= A_{N'}(K')\mu_{N'} \end{aligned} \tag{1}$$

(the subscript represents the cluster size and K is the spin coupling), application of finite-size scaling gives the critical condition

$$A_{N'}(K_c) = A_N(K_c) \tag{2}$$

where K_c is the critical coupling. The thermal eigenvalue is

$$\lambda_T = \left[\left(\frac{d}{dk} A_N\right)\left(\frac{d}{dk} A_{N'}\right)^{-1}\right]\Bigg|_{K_c} \tag{3}$$

and the corresponding critical exponent is

$$y_T = \ln \lambda_T / \ln l \tag{4}$$

where l is the length scaling factor determined by N and N'.

Finite clusters were used in the above-mentioned applications, and recently Plascak and Silva (1986) have extended the method to semi-infinite clusters (i.e. strips).

MFRG gives very good estimates for the critical couplings (the phase diagram) even when the smallest clusters (one- and two-spin clusters) are used. The estimates for the critical indices are, on the other hand, very poor, improving only slightly with increasing cluster size. This is also the case when semi-infinite clusters are used. In this letter I argue that one reason for this is the definition of the length scaling factor l in (4) and that the estimates may be improved by a more judicious choice of l.

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There is clearly some arbitrariness in defining the scaling factor for small clusters. Finite-size scaling is derived for large clusters where a change in the definition of the cluster length changing the cluster length by an amount of the order of one lattice spacing is of no consequence. For small clusters such a change is very important, however. On the other hand, the success of phenomenological scaling (Nightingale 1976, Burkhardt and van Leeuwen 1982) shows that a meaningful definition is possible at least in that case, i.e. for lattice strips.

The definition of the length scaling factor traditionally used in MFRG calculations is

$$l=(N/N')^{1/d} \tag{5}$$

where d is the dimensionality, and N and N' are the number of spins in the two clusters. Or, stating it in another way, the mean length of a N-spin cluster is taken to be

$$L=N^{1/d}. \tag{6}$$

This definition is consistent with the formal connection between a lowest-order cumulant expansion and the MFRG for the isotropic scaling from a hypercube with $N=2^d$ spins to a single spin (Indekeu *et al* 1982). Two objections to the definition (6) can, however, be raised. The first is that, since the cluster is surrounded by a mean field, it simulates a larger cluster than that defined by (6). The second is that, for anisotropic scaling, (6) is inconsistent with the standard procedure in finite-size scaling.

In (6) the length of the cluster is measured in terms of the number of spins. I suggest that it should rather be measured by the number of *interactions*, including the interactions with the surrounding mean field. A single-spin cluster will thus have a length $L=2$. For clusters with different lengths in different directions, such as the two-spin cluster and infinite strips, the form proposed by finite-size scaling is

$$L=\left(\frac{1}{2}\sum_i L_i^{-2}\right)^{-1/2} \tag{7}$$

where the sum runs over the cartesian directions. The form (7) arises naturally from the fact that $1/L$ represents a cutoff in k space (Suzuki 1977) and $(1/L_i)$ must be treated as vectors. Using (7) the length of a two-spin cluster in a hypercubic lattice will be

$$L=6[d/(9d-5)]^{1/2}. \tag{8}$$

Strictly speaking, the finite-size scaling functions are expected, in addition to the length dependence given by (7), to be shape dependent (Ferdinand and Fisher 1969, Barber 1983), so that the MFRG using anisotropic scaling is less well-founded theoretically than the MFRG with isotropic scaling.

A further justification of the scaling factors defined above is obtained by considering the q-state Potts model on a high-dimensional hypercubic lattice. For this model Indekeu *et al* (1982) have shown that the MFRG, using one- and two-spin clusters, gives the critical temperature correctly to second order in $1/d$ when $q=2$:

$$K_c=\frac{1}{2d}+\frac{1}{4d^2}+O\left(\frac{1}{d^3}\right). \tag{9}$$

Table 1. New estimates for the thermal index, y_T.

Model	Type of scaling	N	N'	y_T Old estimate	y_T New estimate	
$d=2$	Isotropic	4	1	0.69	1.18	a
square-lattice Ising		9	4	0.78	1.10	
ferromagnet		16	9	0.82	1.06	
		25	16	0.84	1.03	
	Anisotropic	2	1	0.60	1.28	a
	Infinite strips	2	1	0.66	1.12	b
		3	2	0.74	1.04	
		Exact		1.00		
$d=3$	Isotropic	8	1	0.82	1.40	a
cubic-lattice Ising		27	8	0.95	1.35	
ferromagnet	Infinite strips	2	1	—	1.46	b
		3	2	—	1.60	
		Series		1.587		c
$d=2$	Isotropic	6	3	0.7	0.8	d
triangular-lattice Ising		15	6	0.86	1.0	
antiferromagnet		Exact		1.2		

[a] Indekeu *et al* (1982).
[b] Plascak and Silva (1986) and this work.
[c] Le Gillou and Zinn-Justin (1980).
[d] Slotte (1984).

Using their equation (10) a straightforward calculation of the thermal eigenvalue gives for all q

$$\lambda_T = 1 + \frac{1}{2d} + O\left(\frac{1}{d^2}\right). \tag{10}$$

If one uses the conventional scaling factor (5) the thermal index takes the value $y_T = 1/2 \ln 2 \approx 0.72$, significantly different from the correct (mean-field) value $y_T^{\text{exact}} = 2$. Using the new definition (7) ((8)) one gets the improved estimate

$$y_T = \frac{9}{5} + O\left(\frac{1}{d}\right). \tag{11}$$

This is still not exact, but quite good, bearing in mind that the scaling is extremely anisotropic.

Table 1 shows new MFRG estimates, using (7), for indices calculated previously by various authors. In most cases, including some calculations on other models not included in the table (Plascak 1984a, de Alcantara Bonfim 1984, Plascak and Sá Barreto 1986), the estimates are in general considerably improved.

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